Logic in Action

Chapter 3: Syllogistic Reasoning

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Beyond Propositional Logic

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How to deal with individuals and their properties?

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- The inference involves just three predicates.

Examples

Some mugs are beautiful.
All mugs are useful.

Some useful things are beautiful.

All schools are buildings.
Some schools are tents.

No buildings are tents.

All fruit is nutritious.
All fruit is tasty.

Some tasty things are nutritious.

Some travellers are not caucasian.
None of the tourists is a traveller.

Some tourists are not caucasian.

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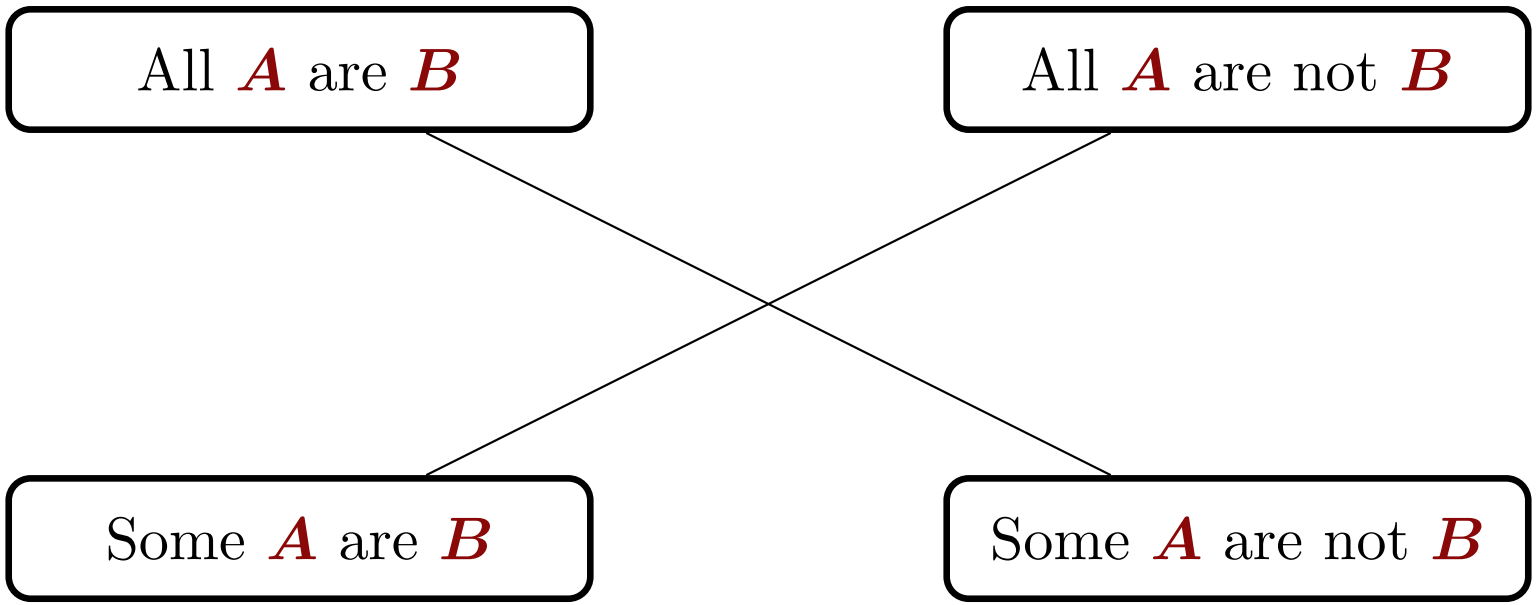

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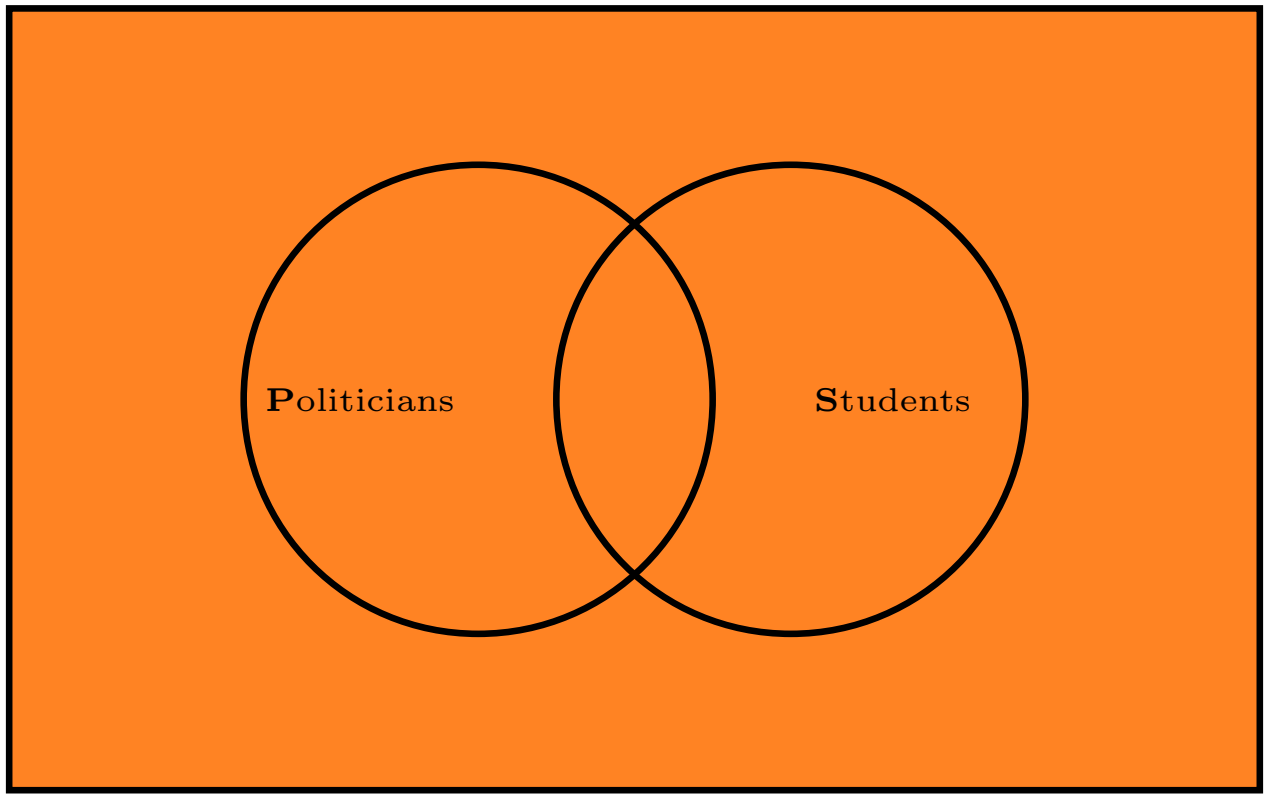
- Usually there is a **domain** U from where the objects are taken from.

$$\{x \in U \mid \varphi(x)\}$$

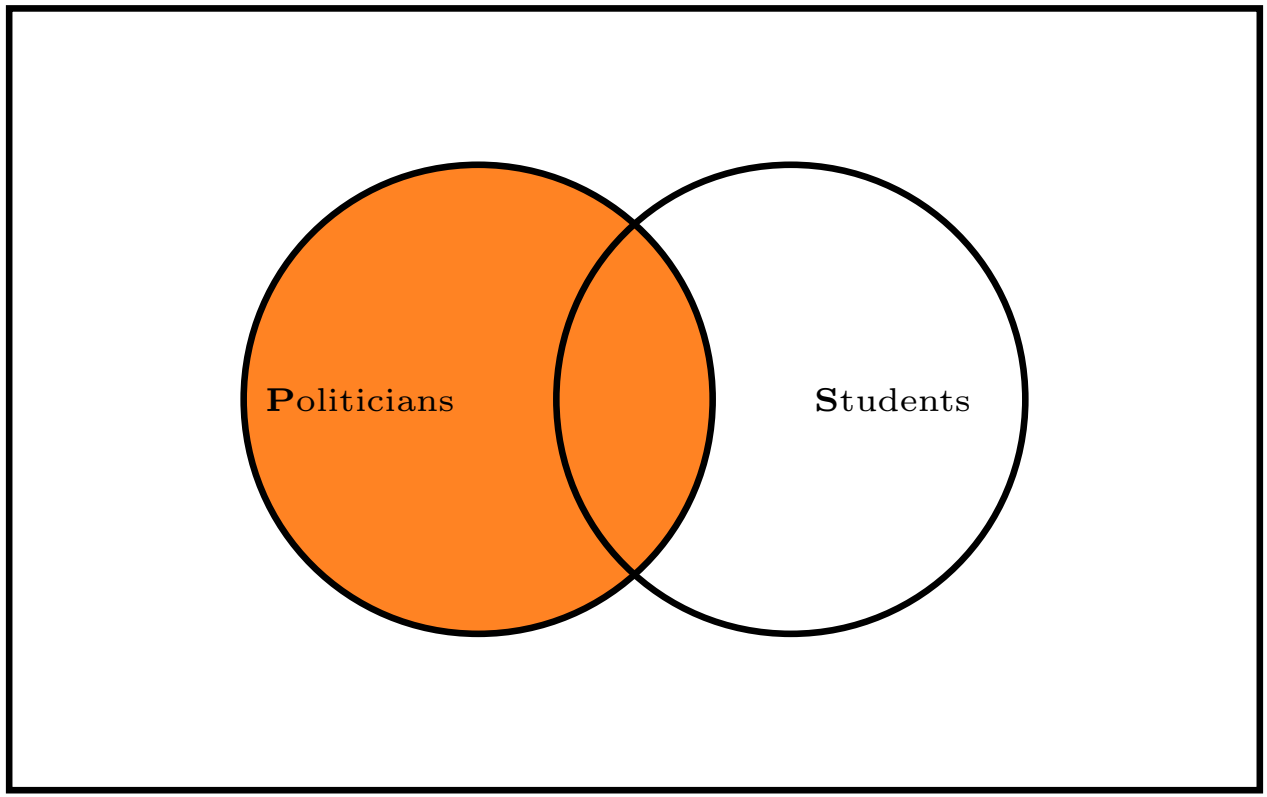
Operations on sets: two predicates

Domain: Humans



Operations on sets: two predicates

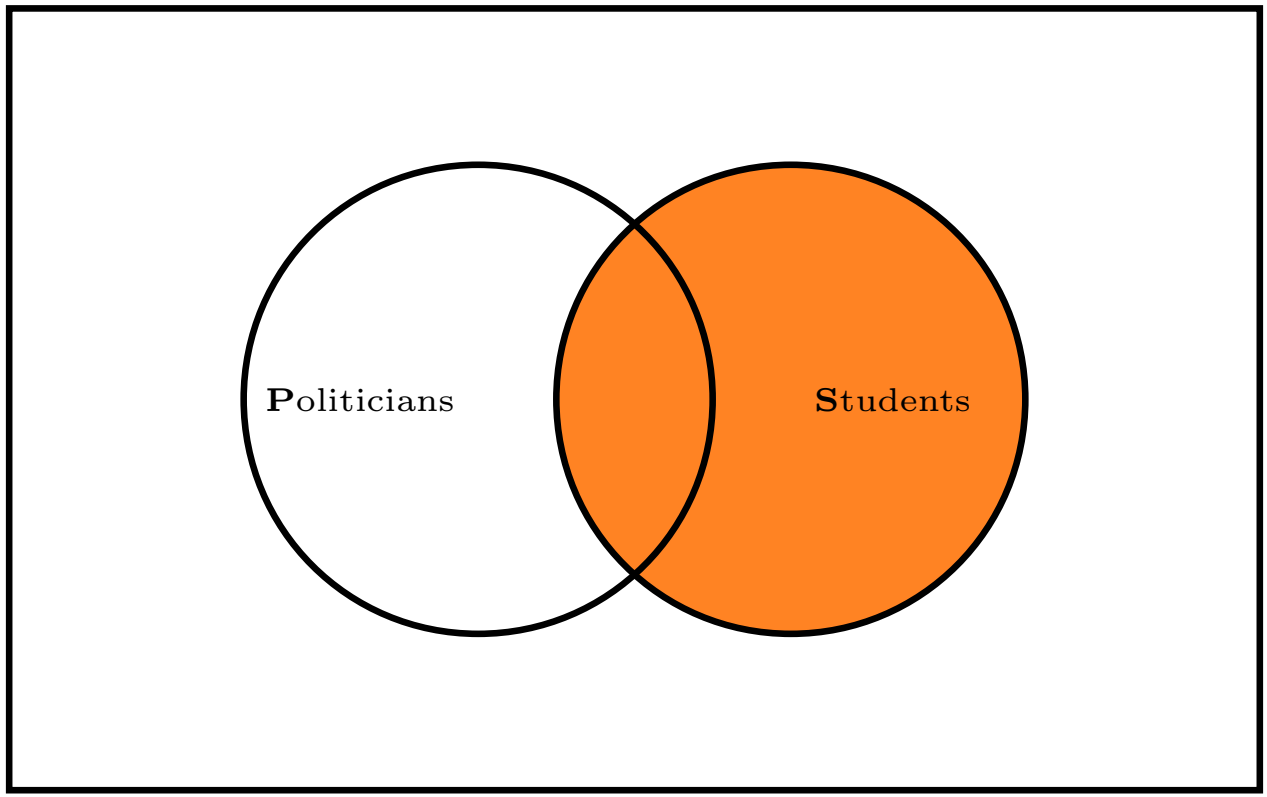
A **set**: **P**oliticians



P

Operations on sets: two predicates

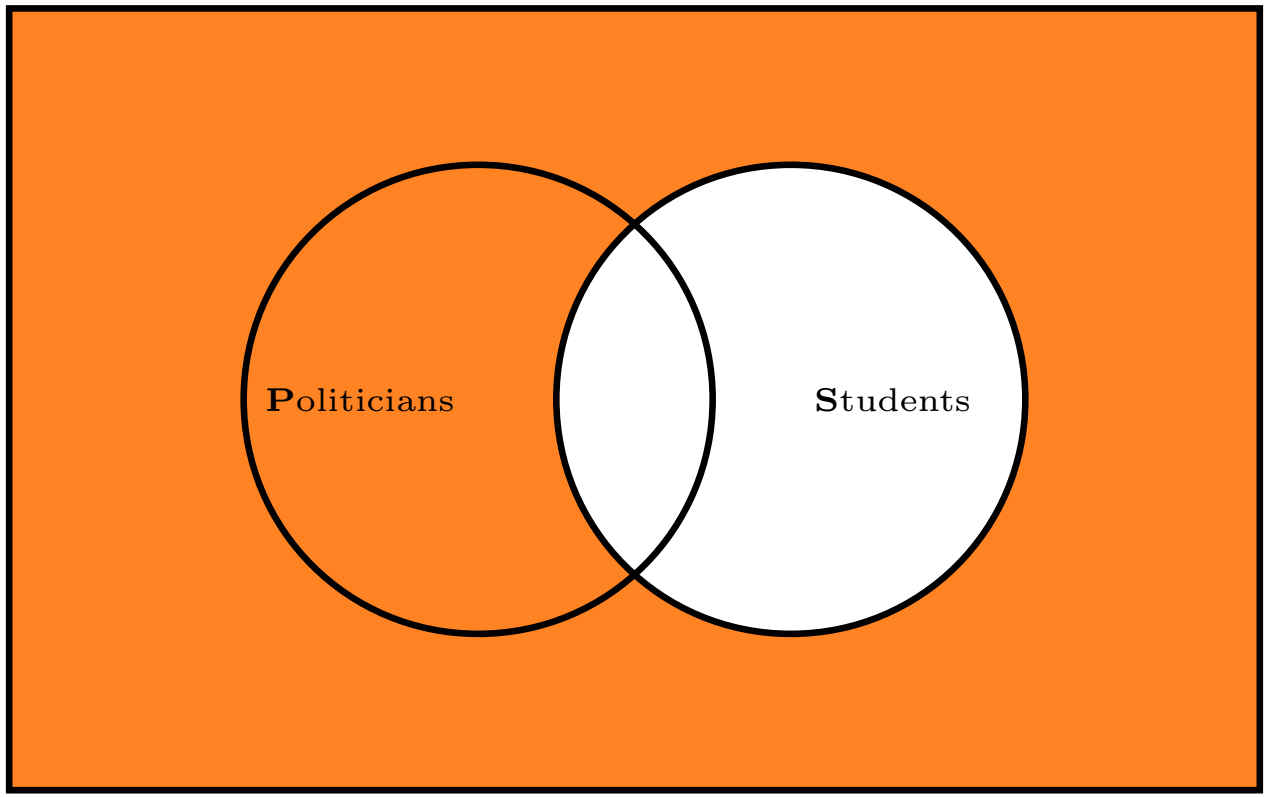
A **set**: **S**tudents



S

Operations on sets: two predicates

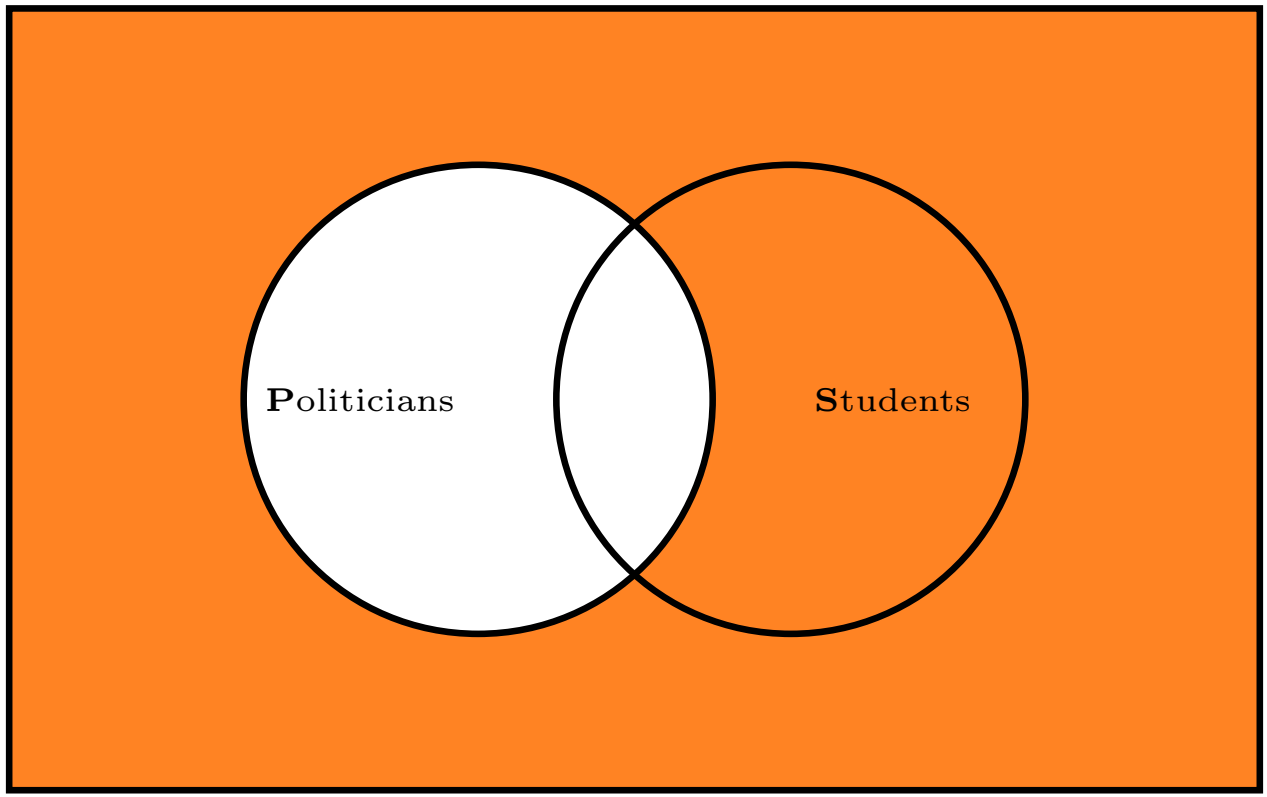
Complement: No Students



$\overline{S}$

Operations on sets: two predicates

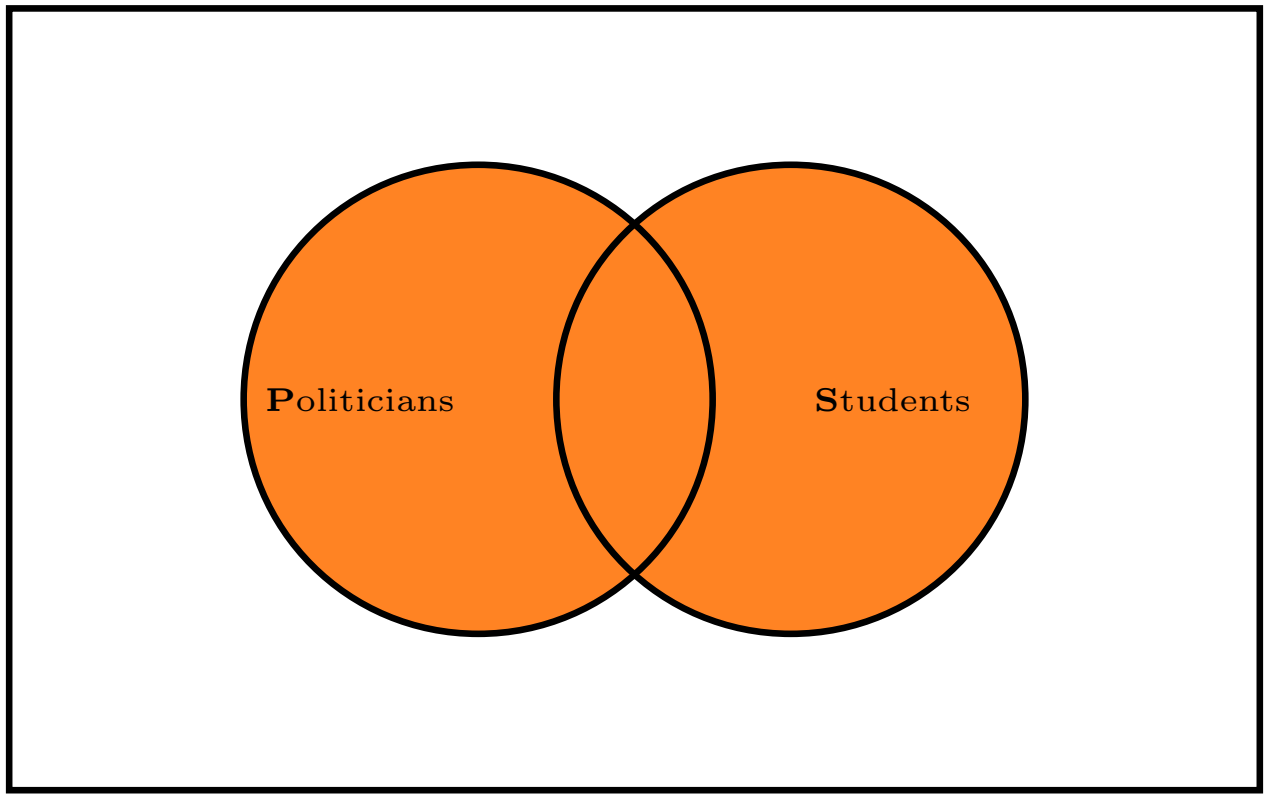
Complement: **No** Politicians



$\overline{P}$

Operations on sets: two predicates

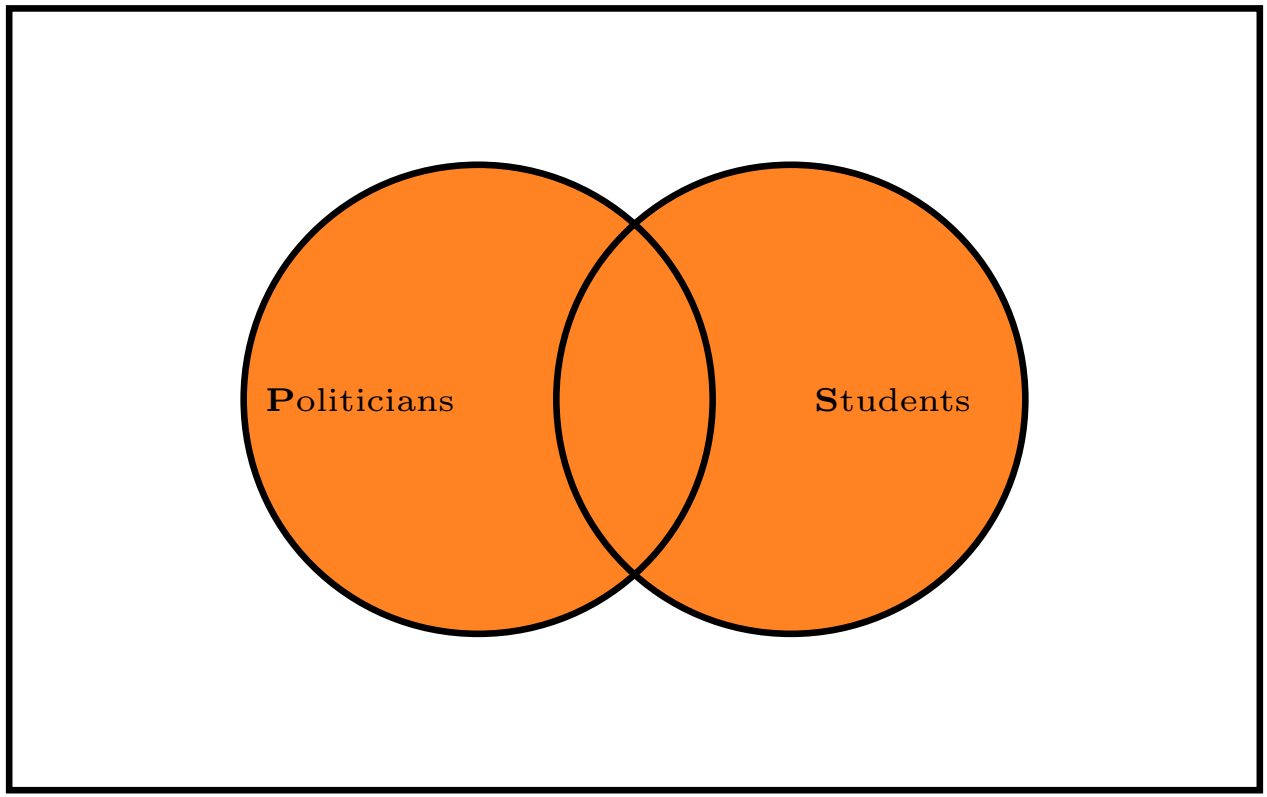
Union: **P**oliticians **or** **S**tudents



$$P \cup S$$

Operations on sets: two predicates

Union: Students **or** Politicians



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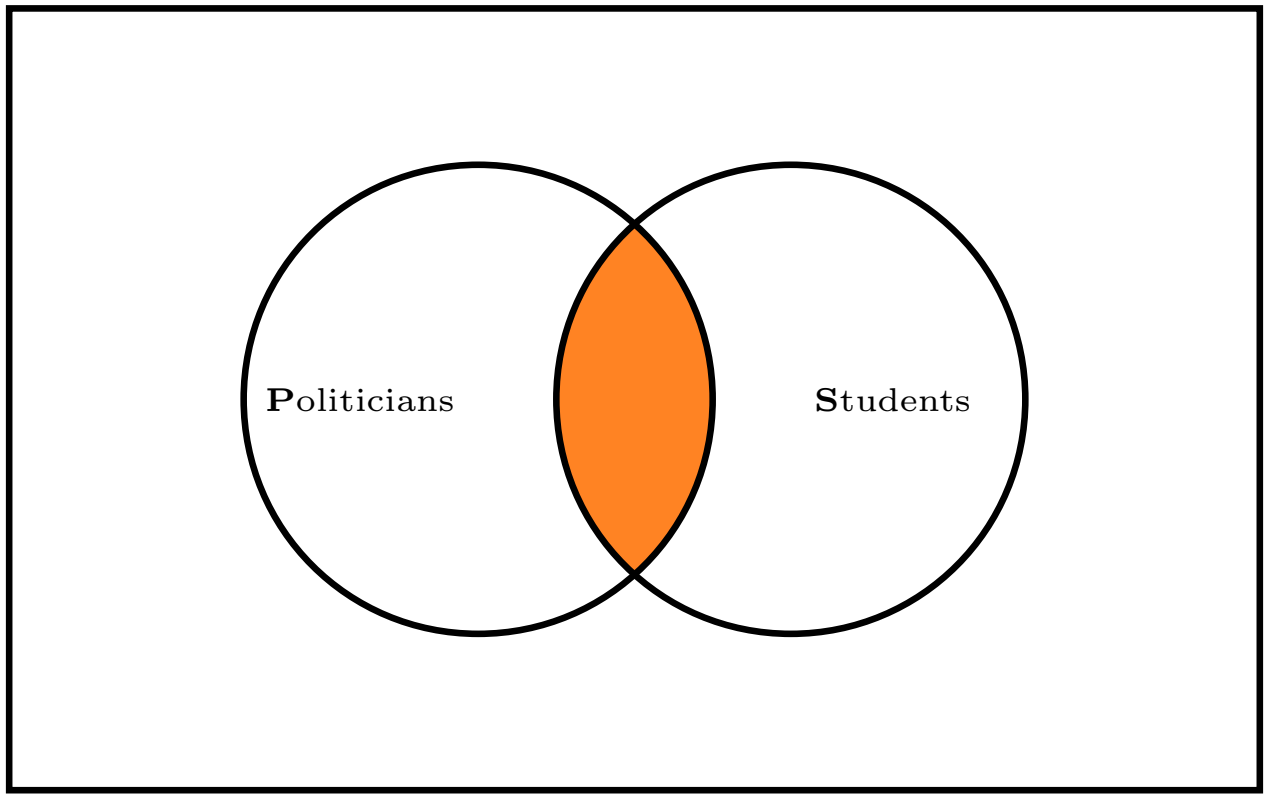
Operations on sets: two predicates

Intersection: **P**oliticians **and** **S**tudents



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Operations on sets: two predicates

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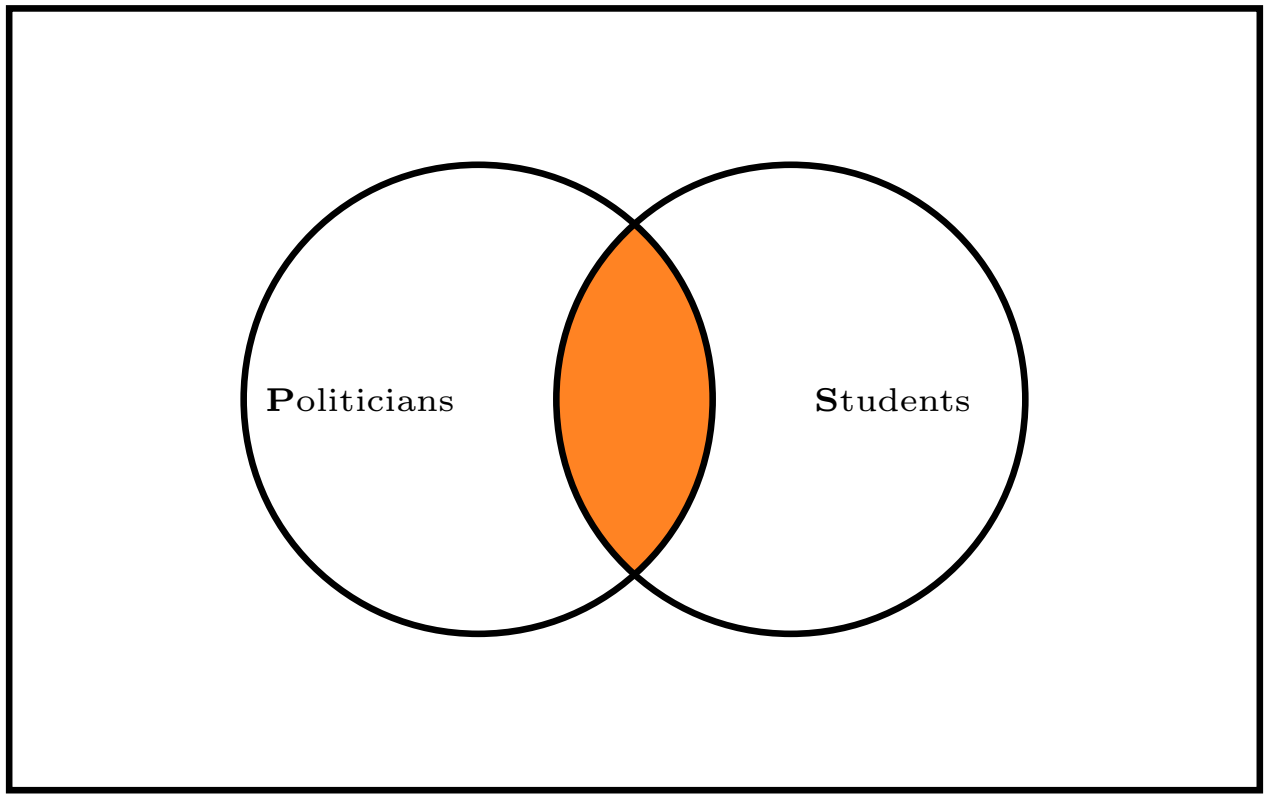
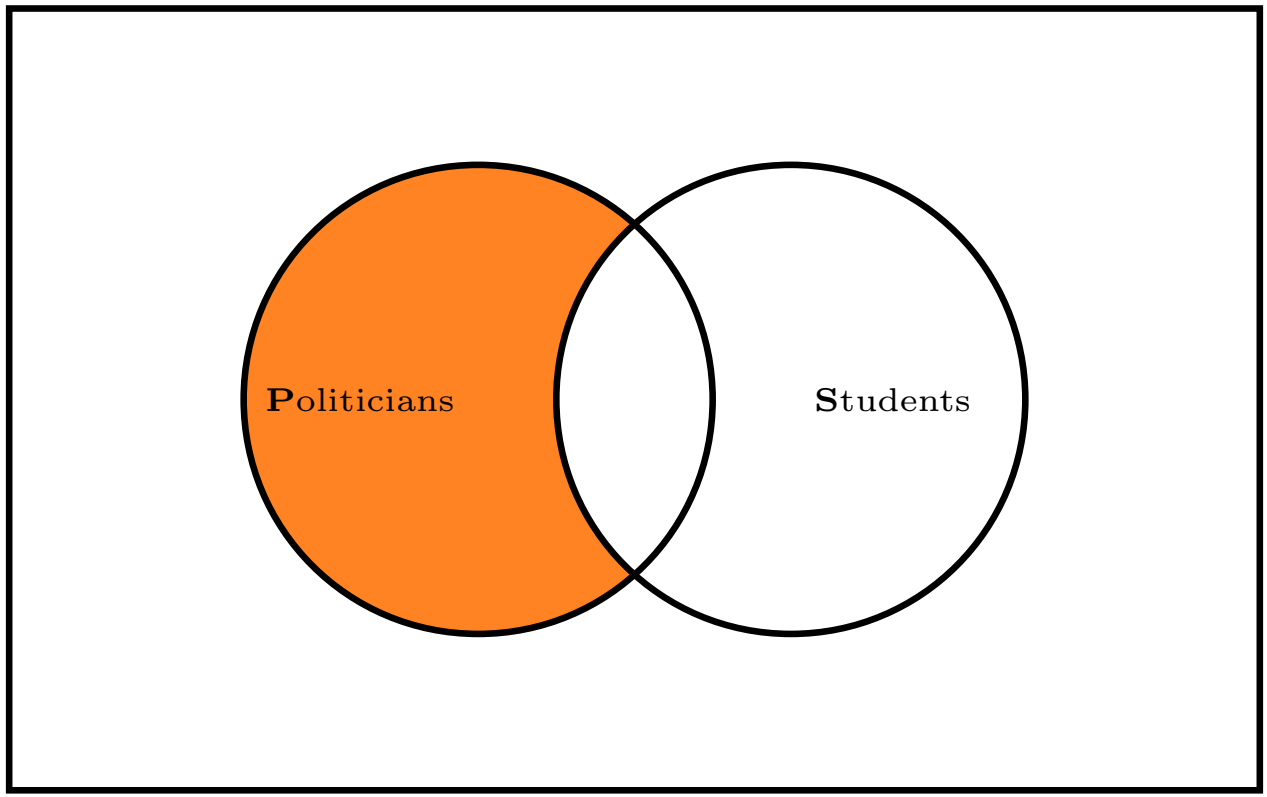


$$S \cap P$$

Operations on sets: two predicates

Difference: Politicians **that are no** Students



$$P \setminus S$$

Operations on sets: two predicates

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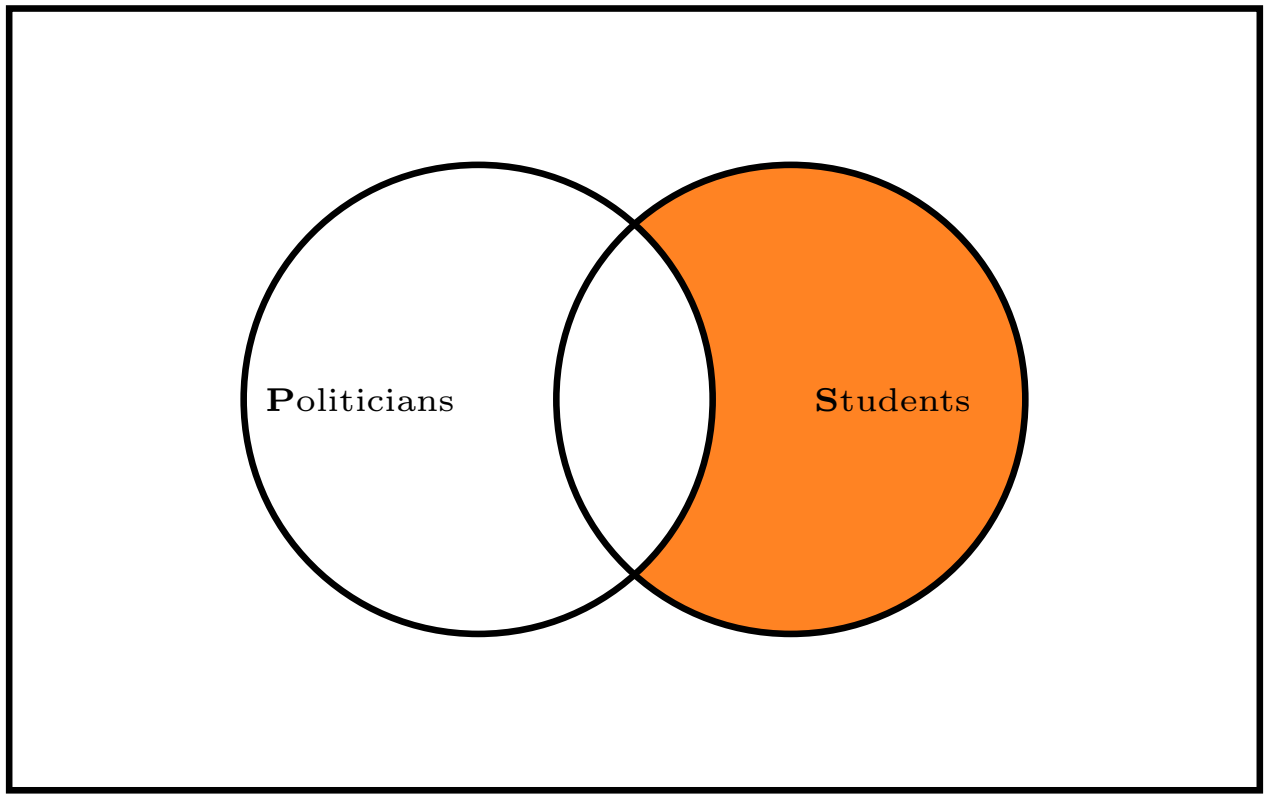
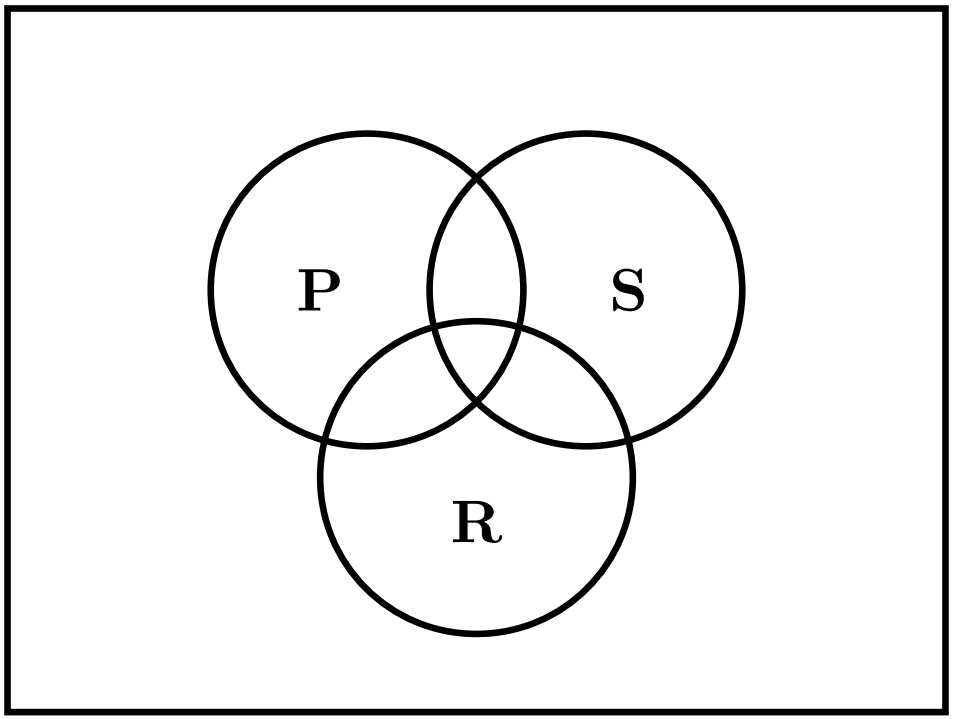


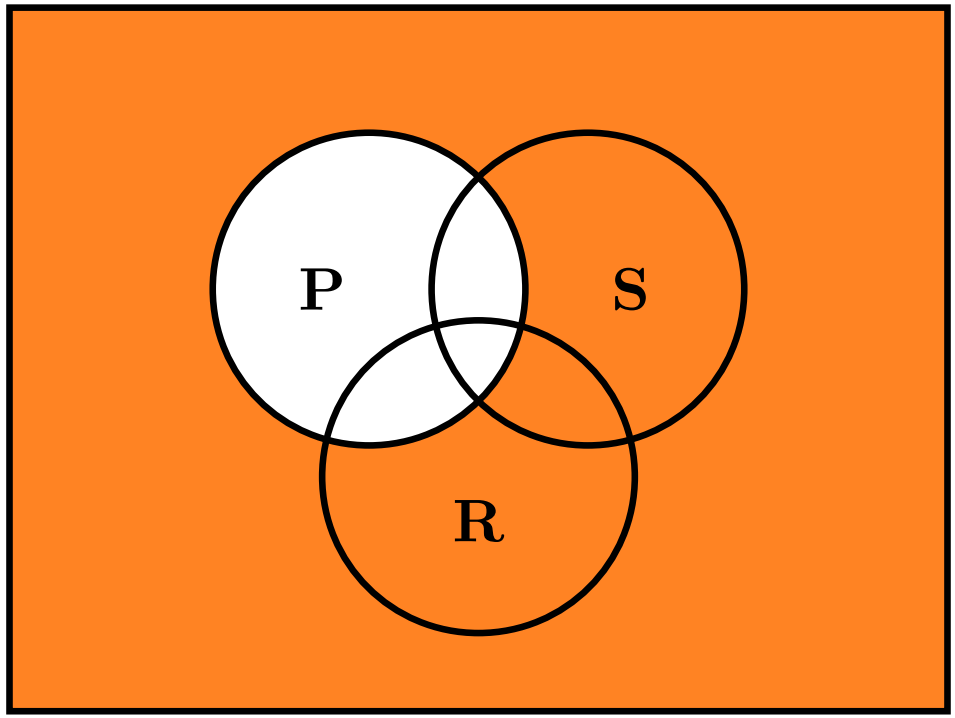
$$S \setminus P$$

Operations on sets: three predicates



All possible combinations are represented in this diagram.

Operations on sets: three predicates



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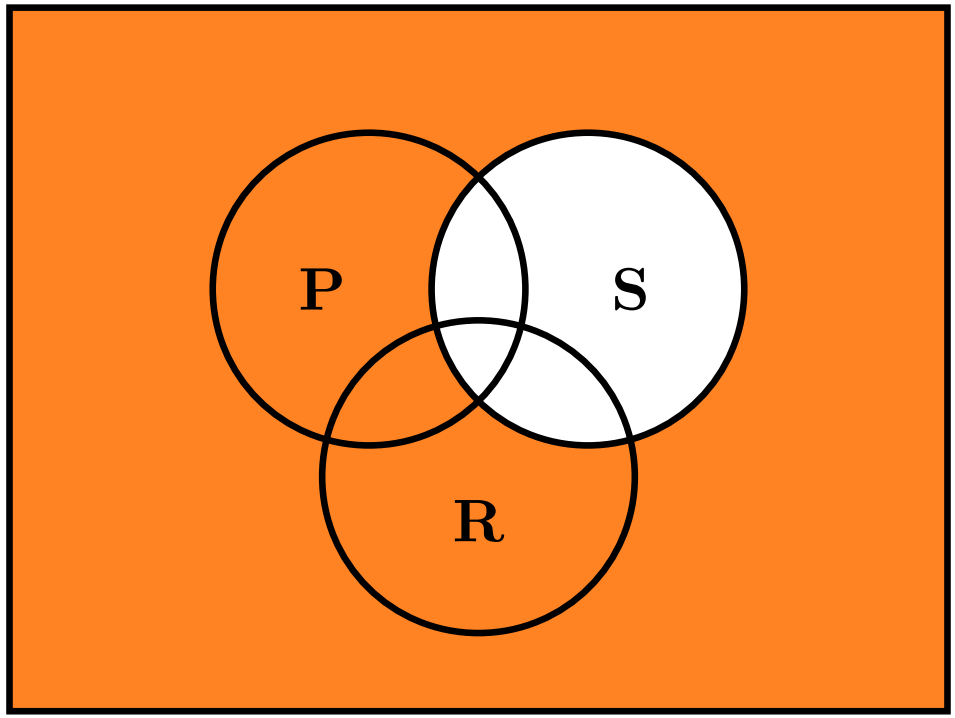
Operations on sets: three predicates



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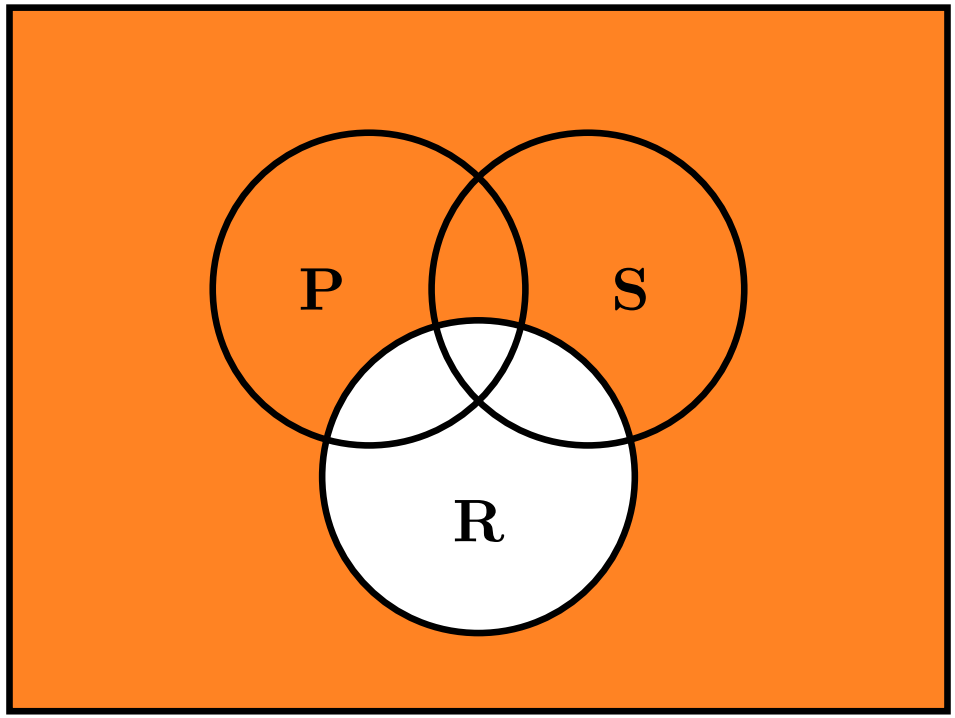
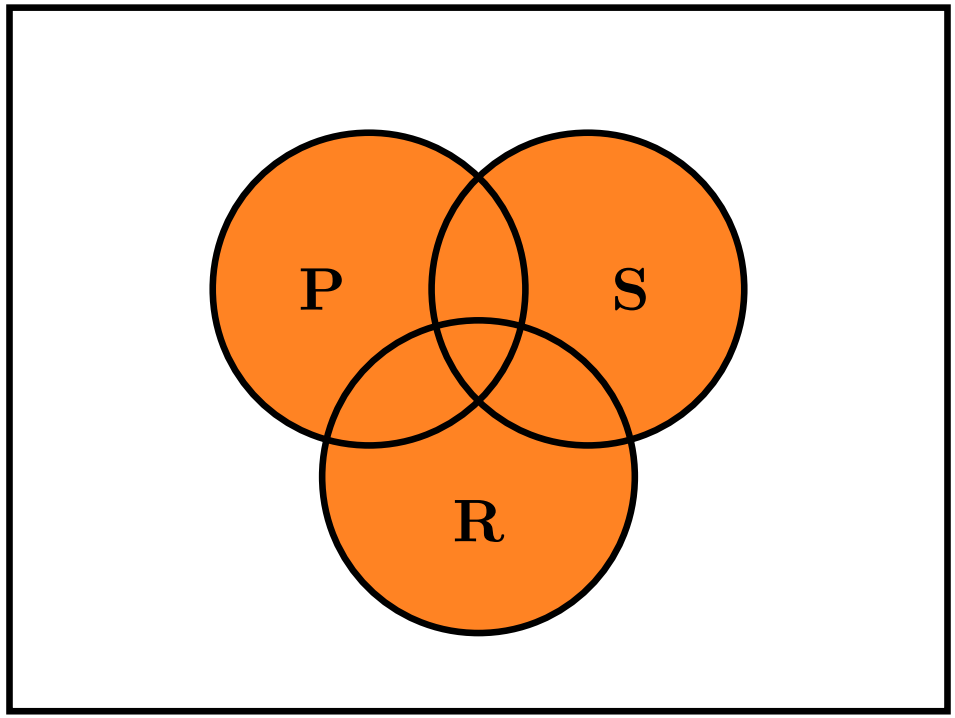


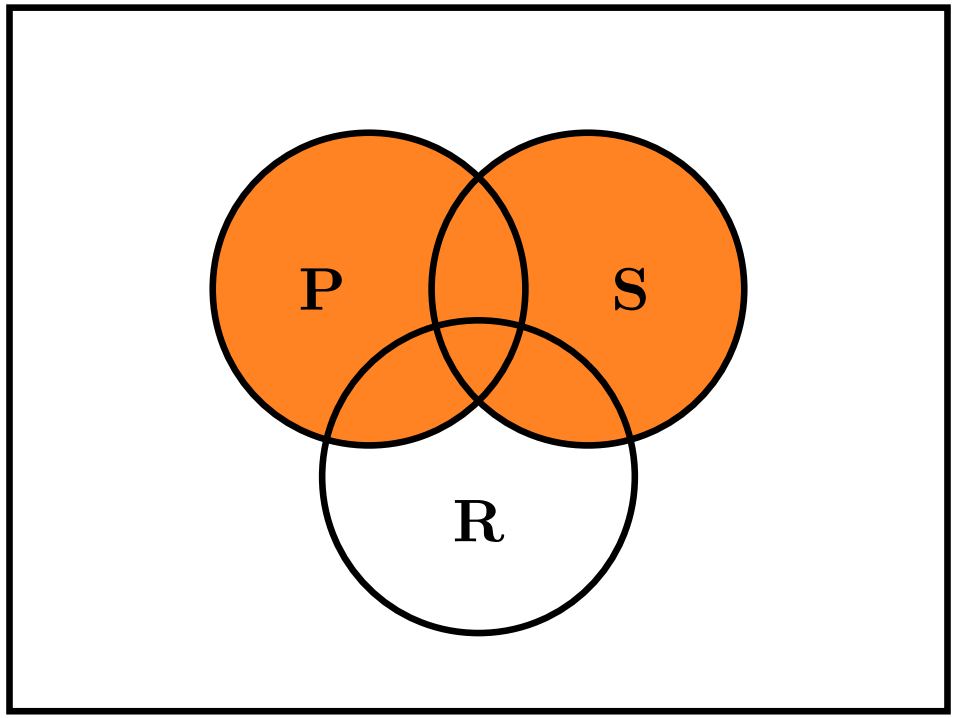
$\overline{R}$

Operations on sets: three predicates



$$P \cup S \cup R$$

Operations on sets: three predicates



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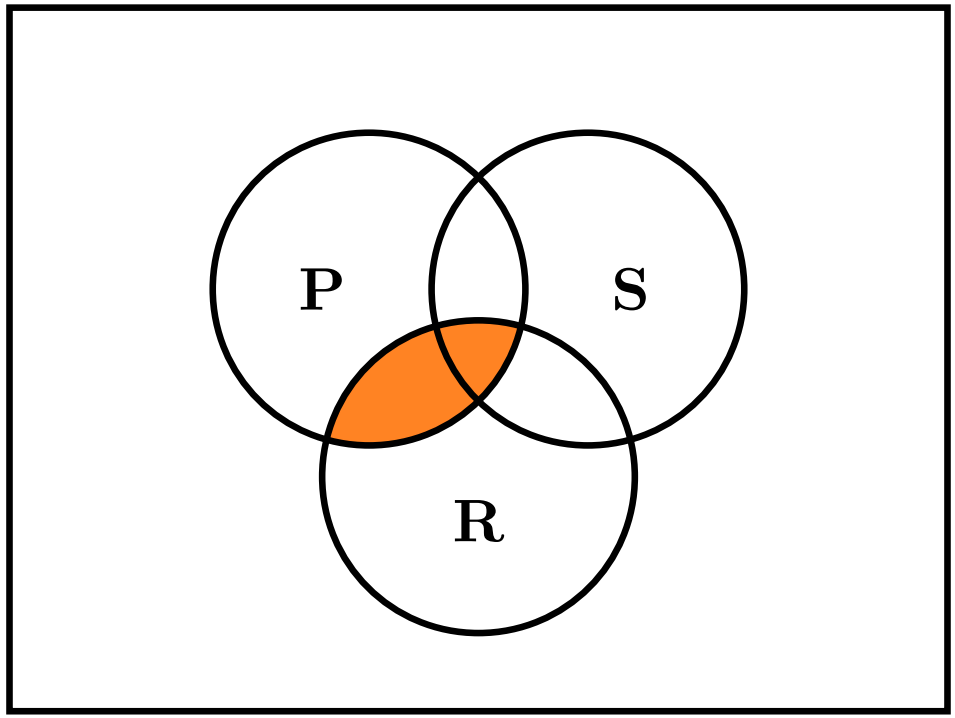
Operations on sets: three predicates



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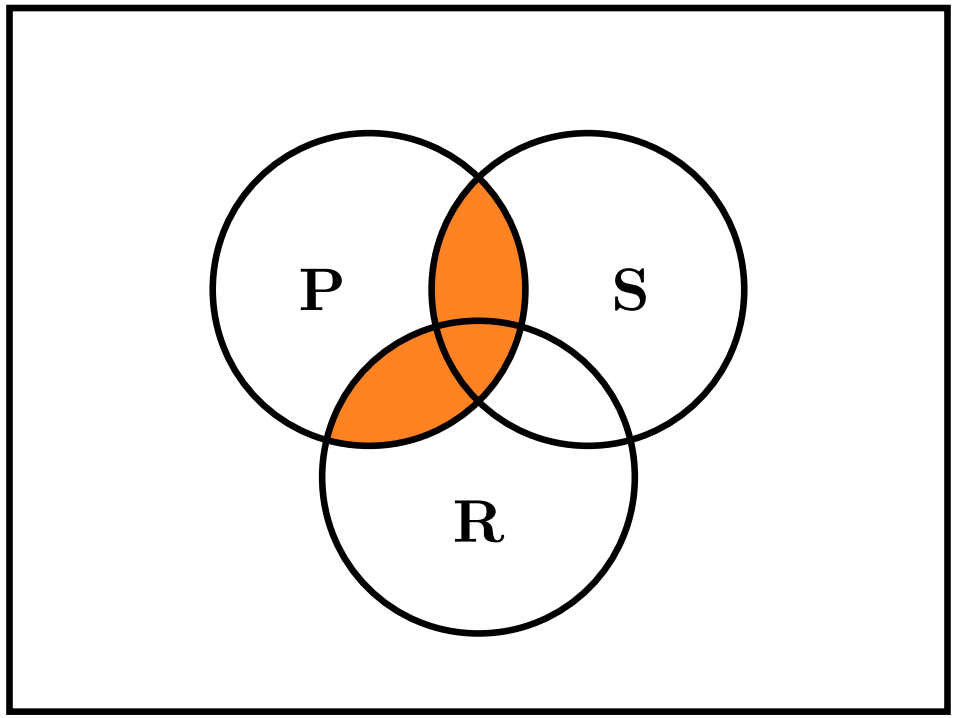

$$P \cap (S \cup R)$$

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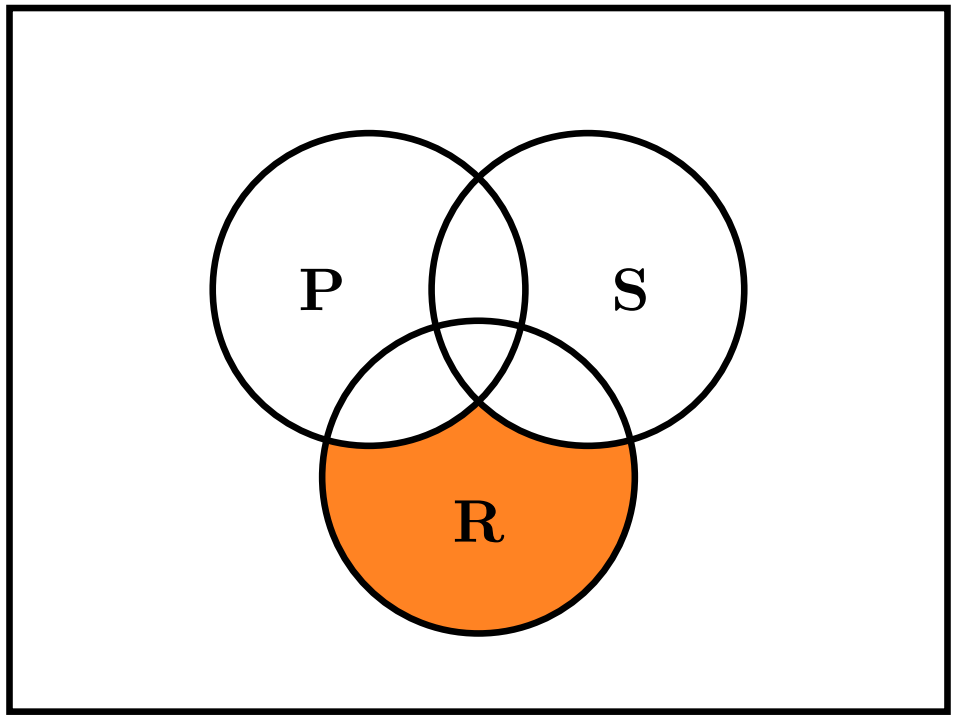
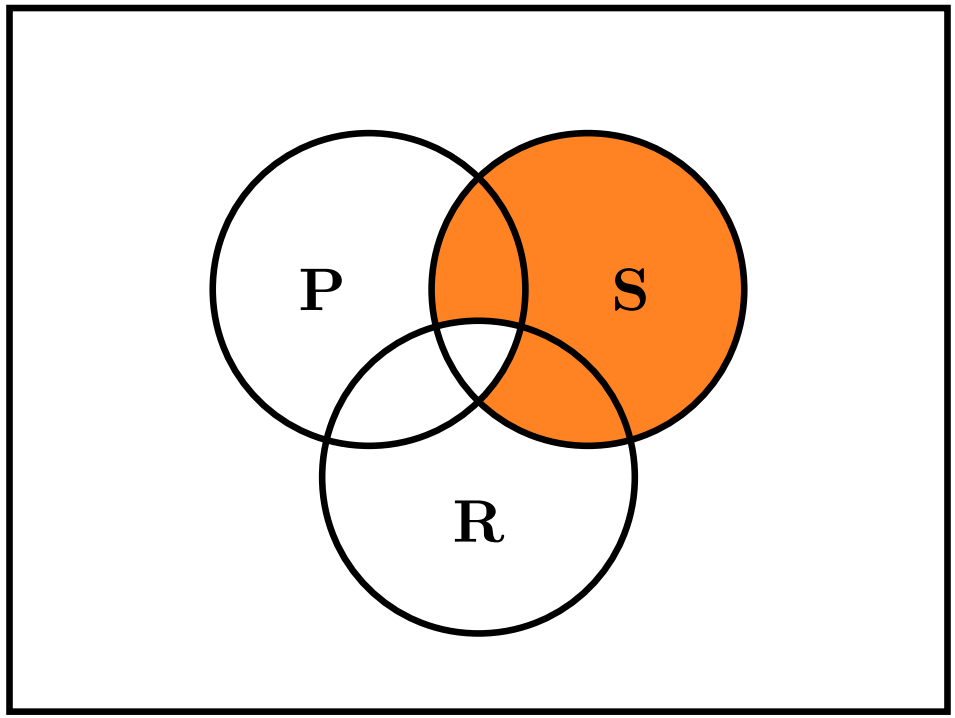


$$R \setminus (P \cup S)$$

Operations on sets: three predicates



$$(S \setminus P) \cup (S \setminus R)$$

How to represent syllogistic situations

- The four statements

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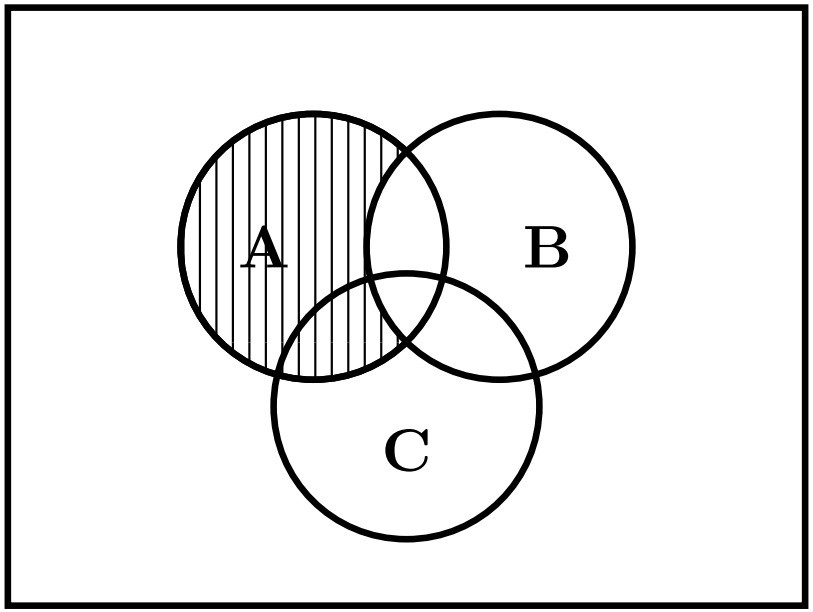
- **All *A* are *B***

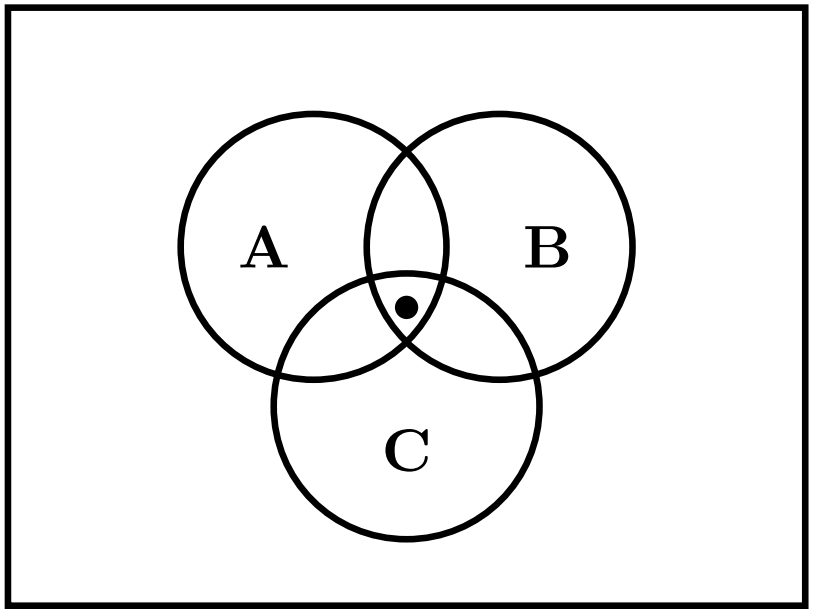
The unique representation of the statement:

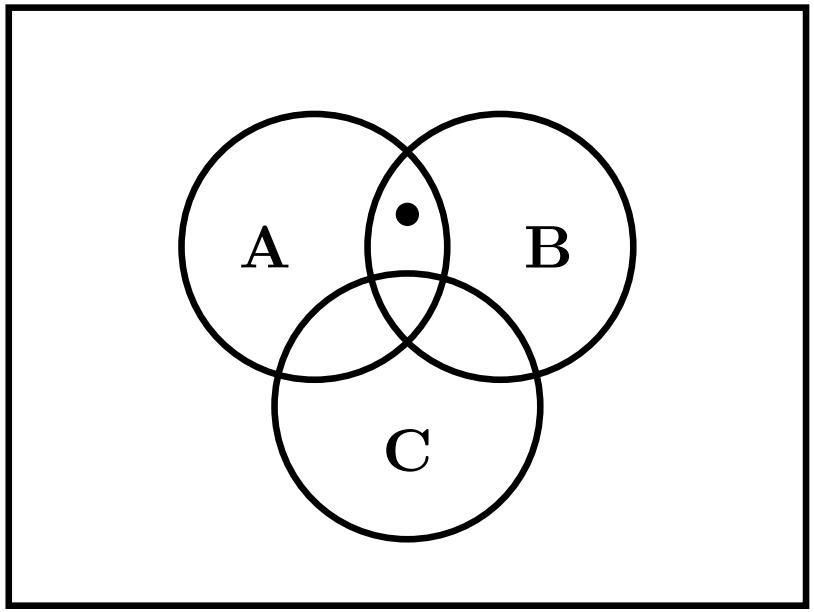


How to represent syllogistic situations

- **Some *A* are *B***

The two representations of the statement:

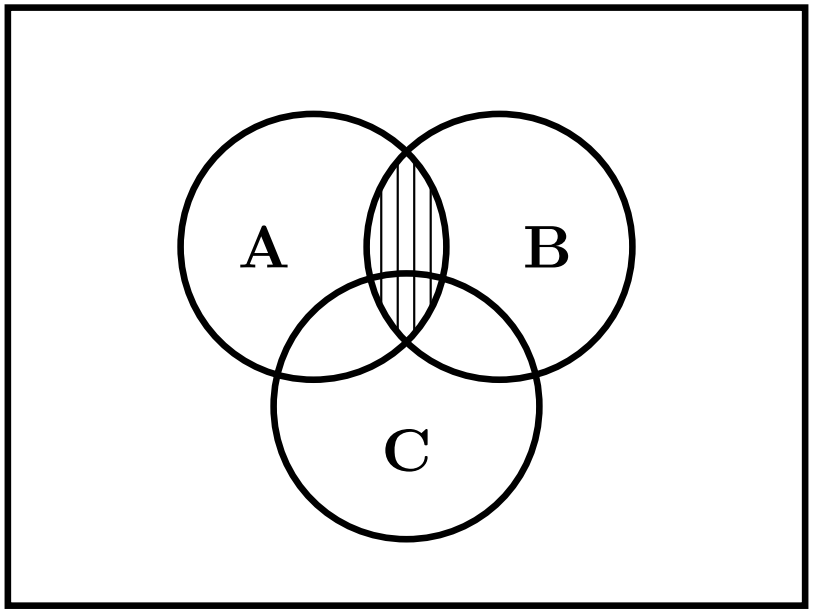




How to represent syllogistic situations

- **All *A* are not *B*** (**No *A* is *B***)

The unique representation of the statement:



How to represent syllogistic situations

- **Some *A* are not *B* (Not all *A* are *B*)**

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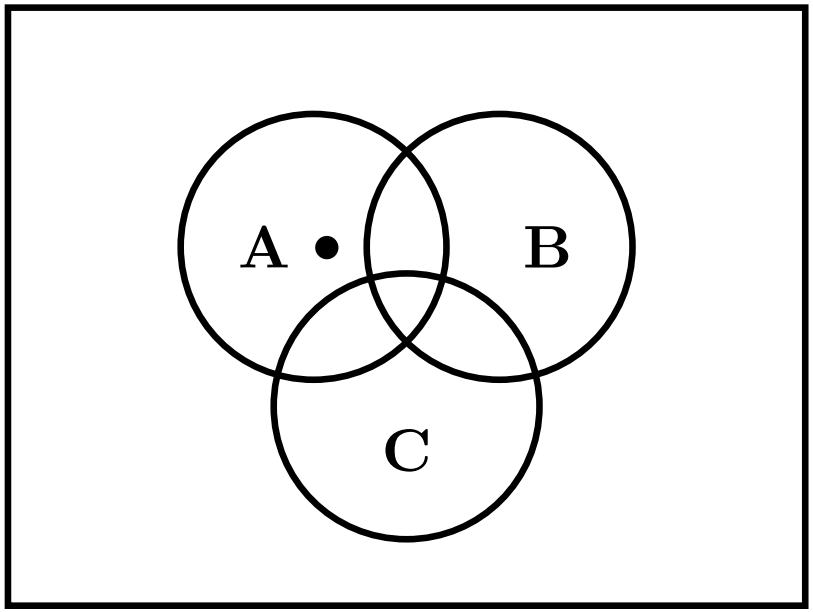

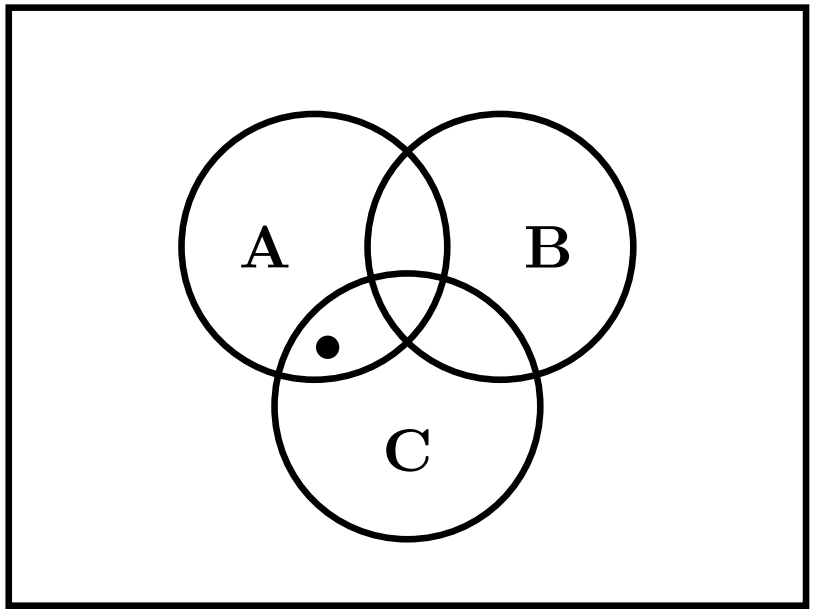

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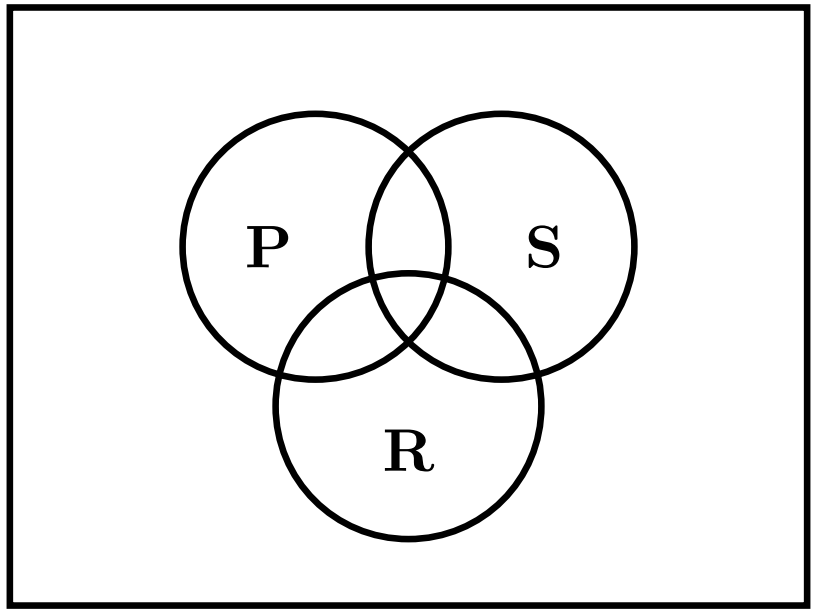
4. **Check conclusion** Verify that **at least one** of the conclusion's representation is in **all** the diagrams.

The syllogistic method: example (1)

	All politicians are rich.	**To draw**
?	No student is politician.	**To draw**
	No student is rich.	**To verify**



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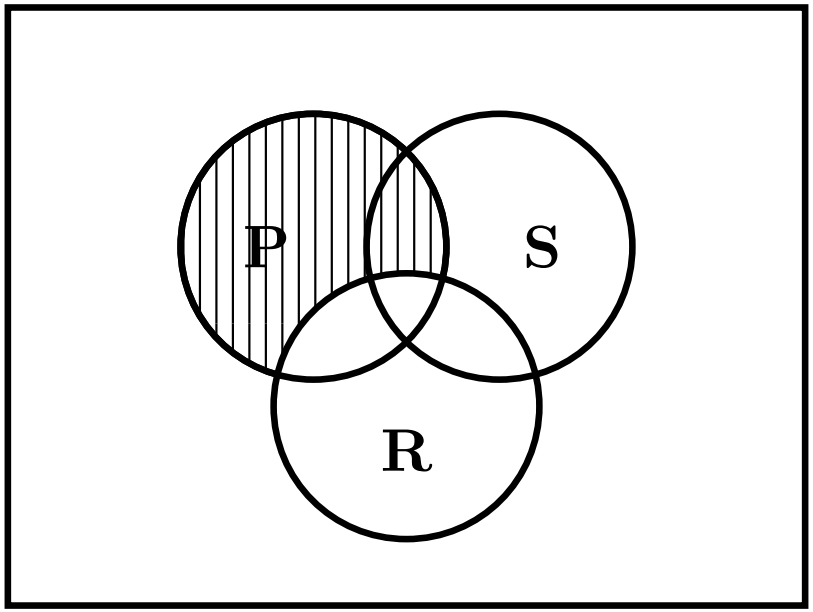

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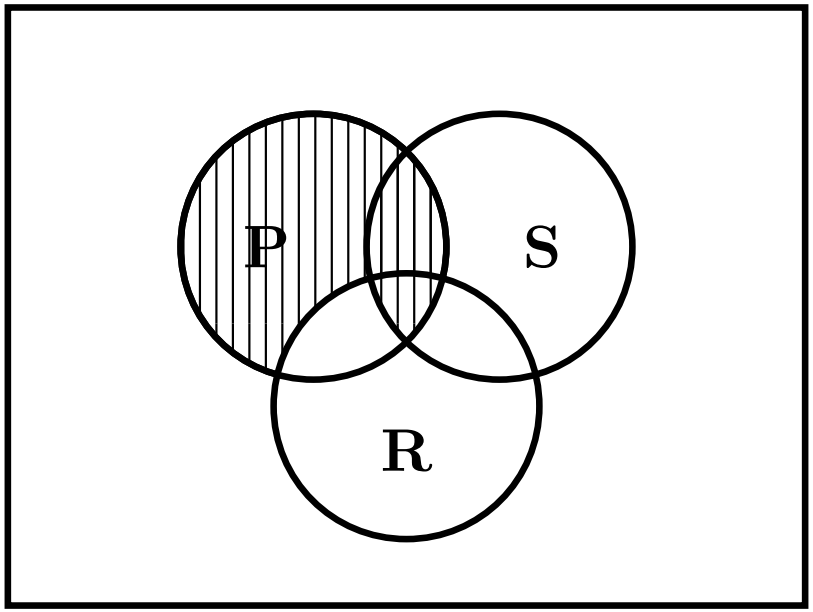

The syllogistic method: example (1)

	All politicians are rich.	**Drawn**
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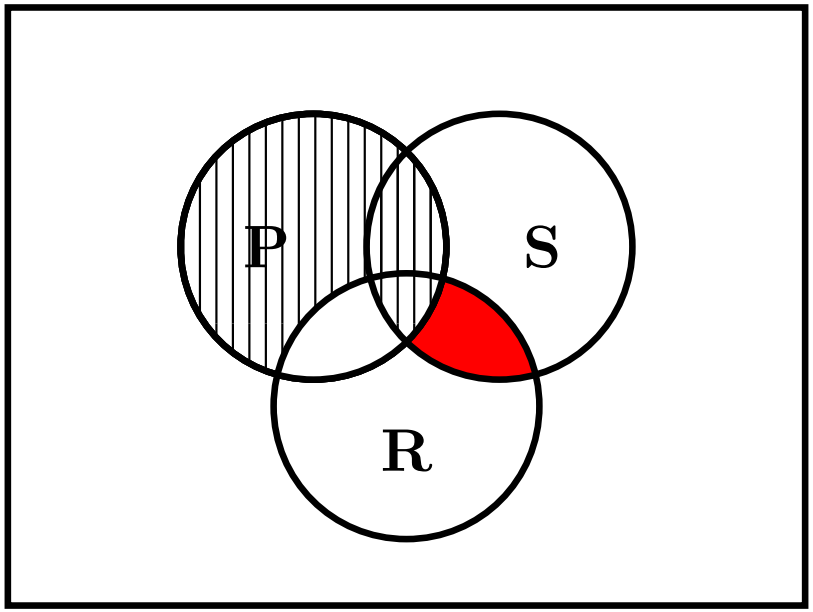
	No student is rich.	**Fail**



The unique conclusion's representation **is not** in the unique diagram.

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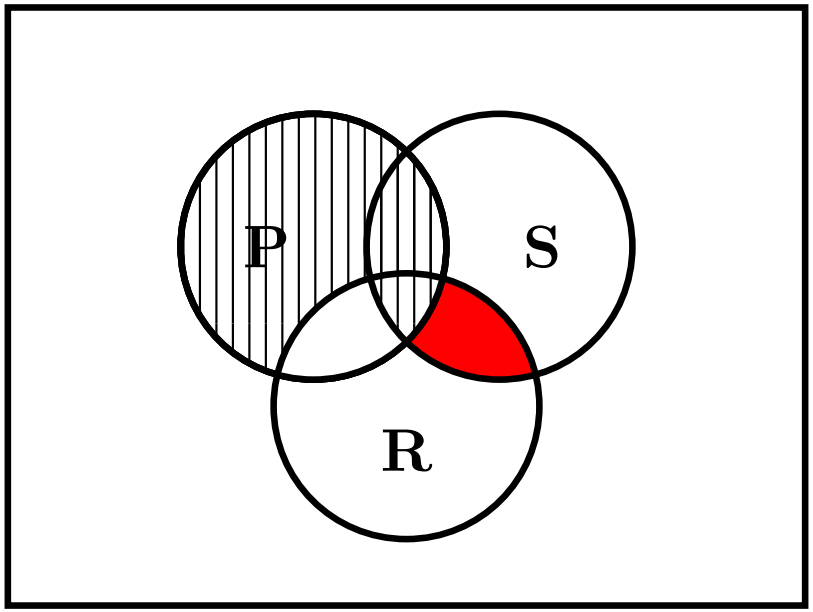

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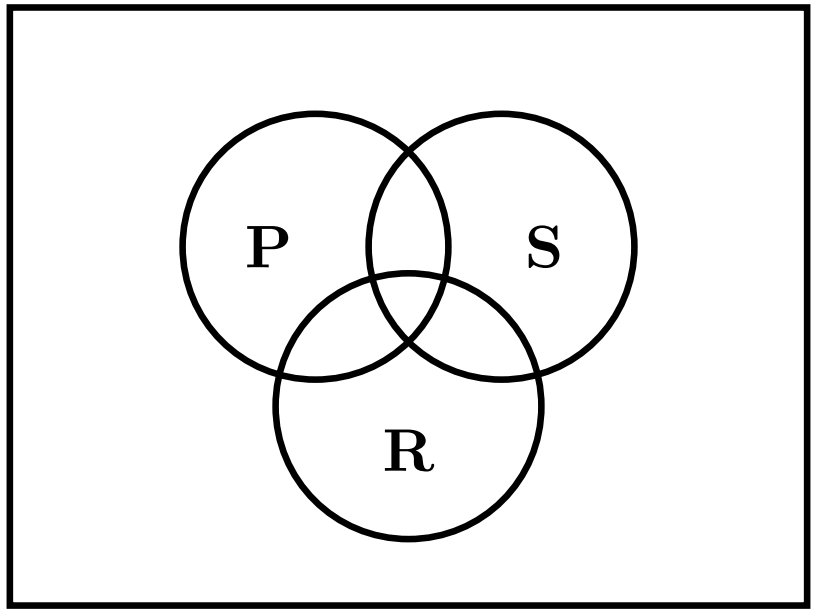
The syllogistic method: example (2)

	All students are politician.	**To draw**
	All politician are rich.	**To draw**
?	---	
	All students are rich.	**To verify**



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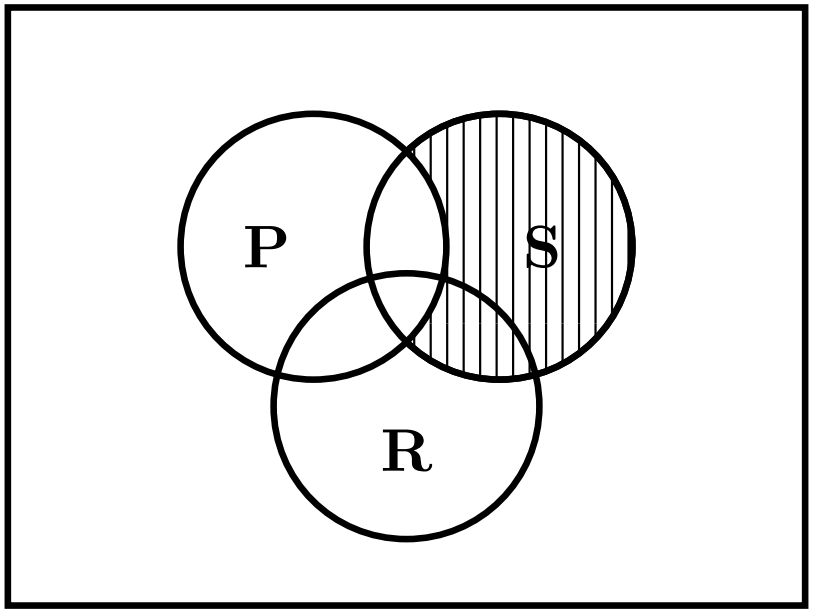

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	All students are politician.	**Drawn**
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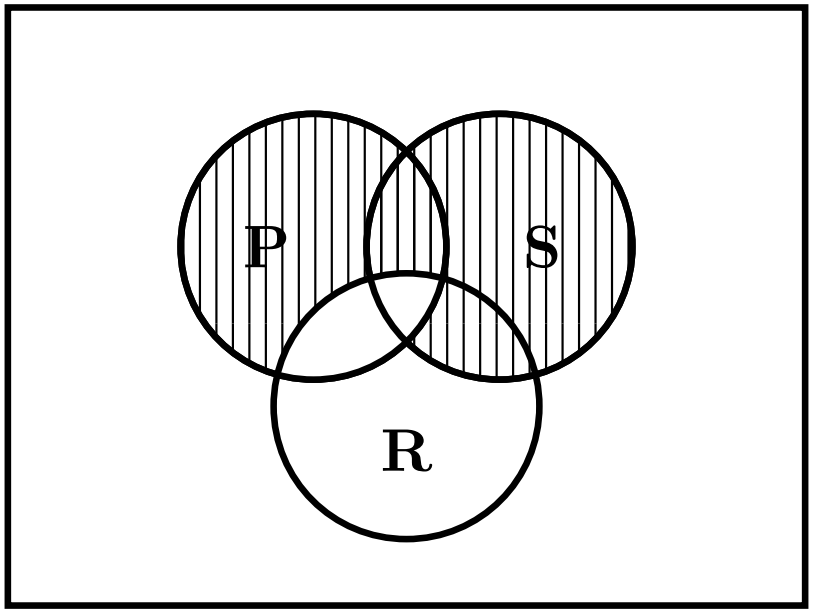
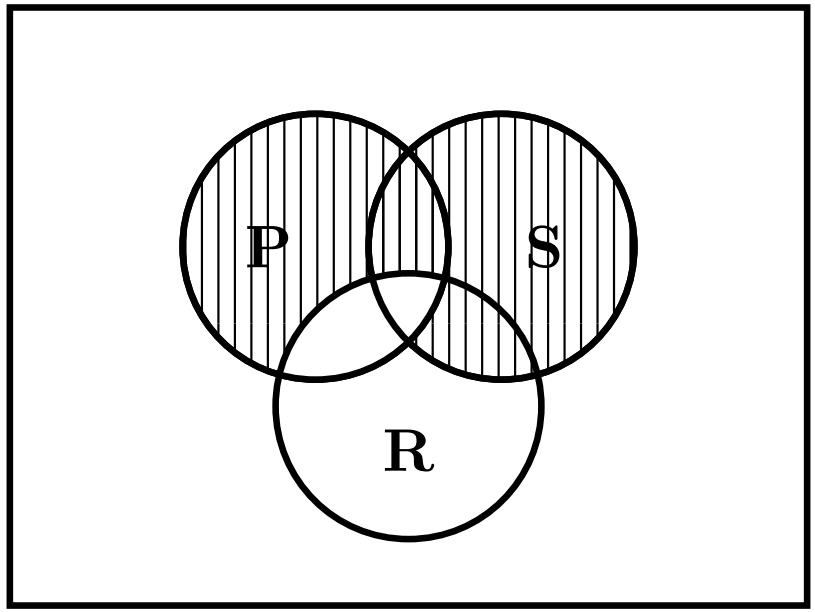


The syllogistic method: example (2)

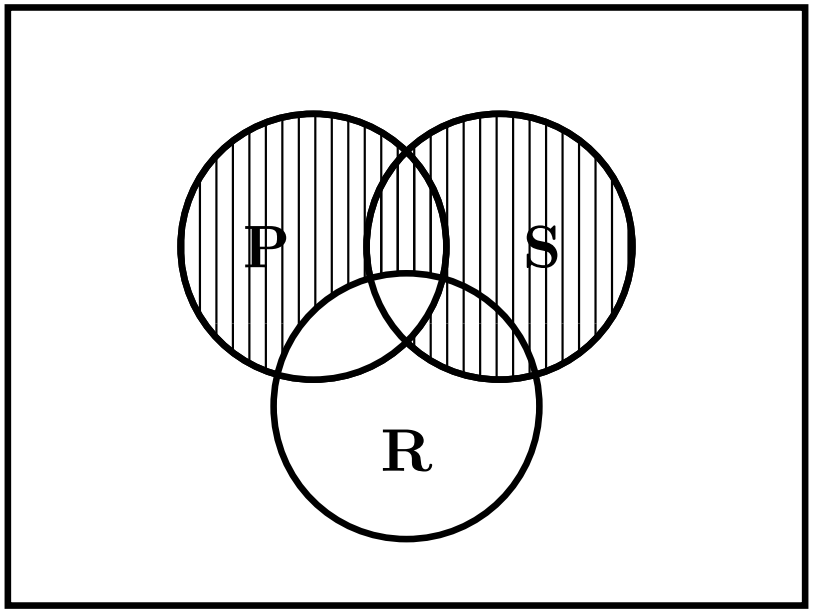
	All students are politician.	**Drawn**
?	All politician are rich.	**Drawn**
	All students are rich.	**Success**



The unique conclusion's representation **is** in the unique diagram.

The syllogistic method: example (2)

	All students are politician.	**Drawn**
✓	All politician are rich.	**Drawn**
	All students are rich.	**Success**



The unique conclusion's representation **is** in the unique diagram.

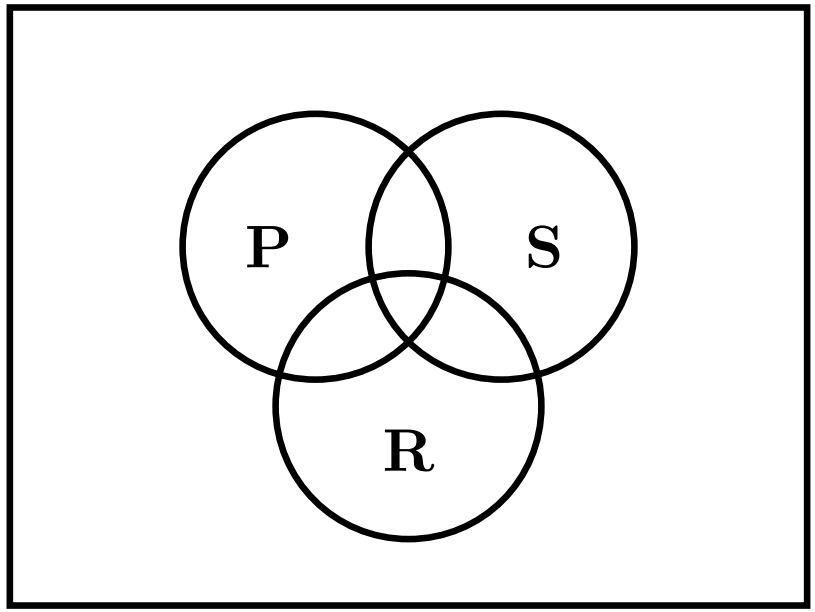
The syllogistic method: example (3)

All students are rich. **To draw**

Some students are politicians. **To draw**

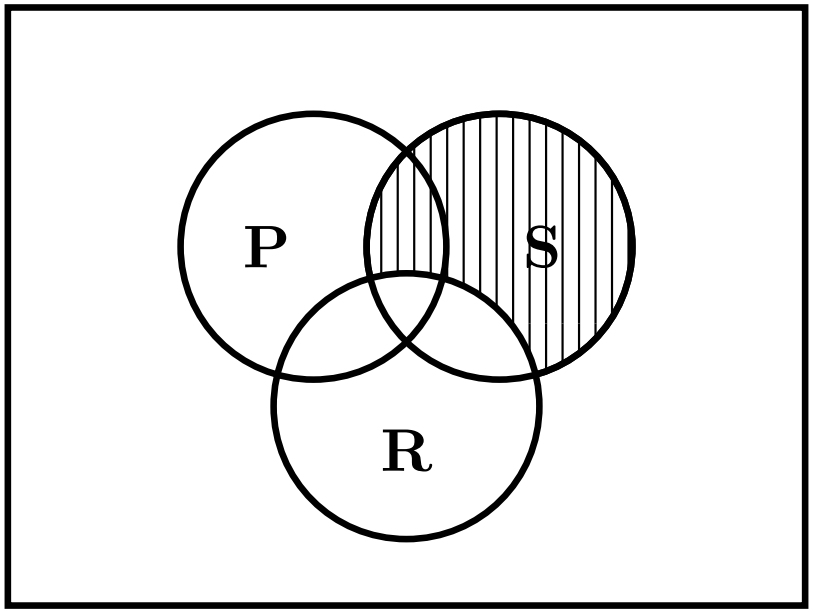
? ——————————

Some students are rich. **To verify**



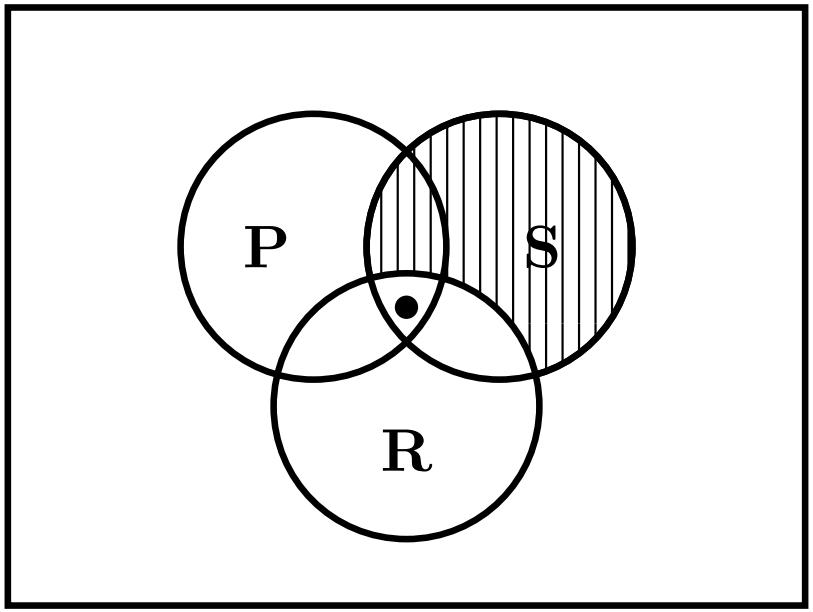
The syllogistic method: example (3)

	All students are rich.	**Drawn**
?	Some students are politicians.	**To draw**
	Some students are rich.	**To verify**



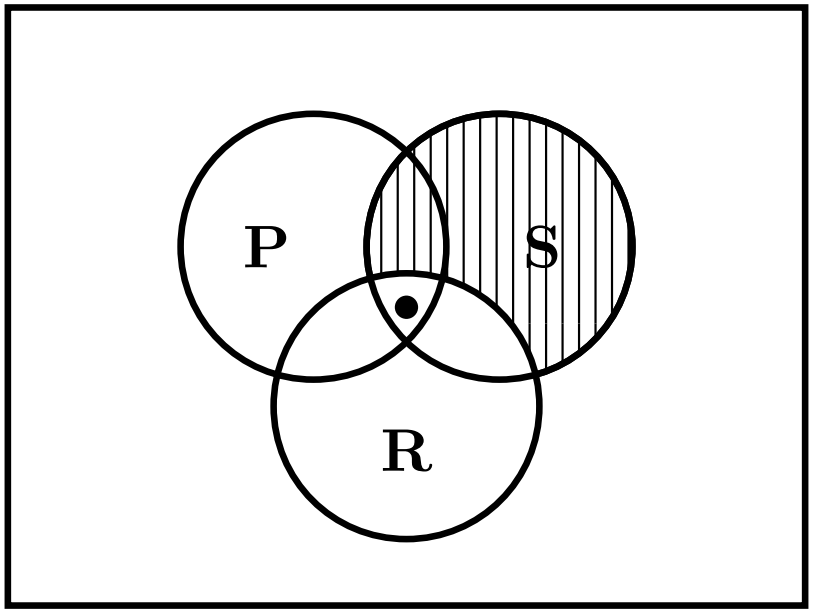
The syllogistic method: example (3)

	All students are rich.	**Drawn**
?	Some students are politicians.	**Drawn**
	Some students are rich.	**To verify**



The syllogistic method: example (3)

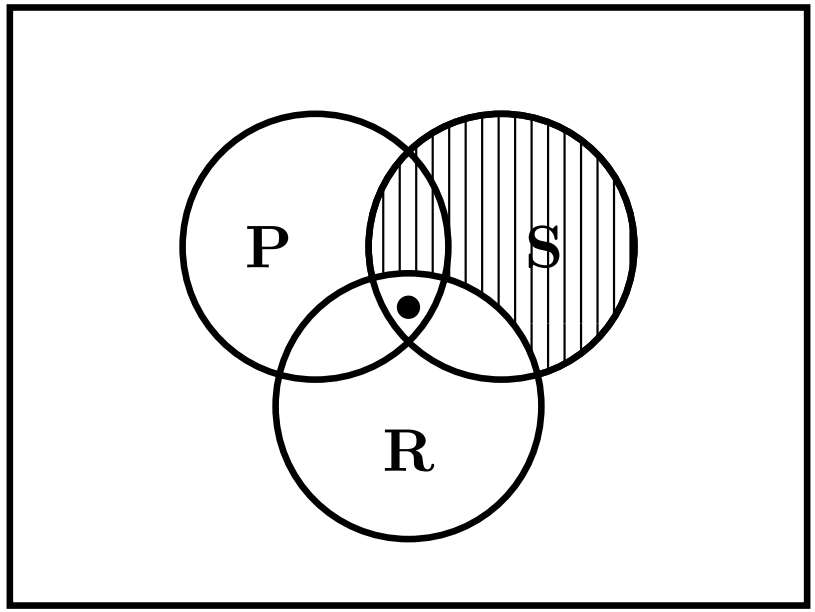
	All students are rich.	**Drawn**
?	Some students are politicians.	**Drawn**
	Some students are rich.	**Success**



One of the conclusion's representation **is** in the unique diagram.

The syllogistic method: example (3)

	All students are rich.	**Drawn**
✓	Some students are politicians.	**Drawn**
	Some students are rich.	**Success**



One of the conclusion's representation **is** in the unique diagram.